Convection induced by an inclined temperature gradient in a shallow horizontal layer

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A reexamination is made of convection, induced by an applied temperature gradient inclined to the vertical, in a shallow horizontal layer. The horizontal component of this gradient induces a Hadley circulation, which becomes unstable when the vertical component is sufficiently large. A linear stability analysis is carried out, for the case of rigid conducting horizontal boundaries, in terms of horizontal and vertical Rayleigh numbers, R_H and R_V, respectively. This analysis is valid for any Prandtl number, Pr, and incorporates the case, $R_V = 0$. The differential equation system is solved using a direct Galerkin approximation. This is convenient for numerical calculations in the parameter range of interest and also enables some general results to be obtained analytically. The results confirm and extend those of previous investigators.

Keywords: inclined temperature gradient; thermal instability

Introduction

Many articles have been written on convection, in a horizontal fluid layer, induced by imposed temperature gradients that are either vertical or horizontal, whereas few articles have dealt with convection induced by an inclined temperature gradient. The available evidence (e.g., in the articles cited later) suggests that the differential equation system that governs such a flow will, in general, have multiple solutions, and so a direct numerical approach is not guaranteed to find the solution that is physically significant. Thus, it is of interest to examine in detail a simple situation that is likely to be paradigmatic for more complicated problems. Besides this theoretical importance, the configuration examined later is useful for modeling a number of physical systems. Drummond and Korpela (1987) give references to studies of the region enclosing auxiliary cooling systems for high-temperature gas-cooled reactors, circulations in planetary atmospheres, dispersion of pollutants in estuaries, the growth of metal crystals, and a solar-energy storage system.

Accordingly, we consider flow in a shallow horizontal box (one with small height-to-length and height-to-width aspect ratios). The unicellular flow induced in such a box by an imposed lengthwise temperature gradient has been called the Hadley circulation by Hart (1972). Well away from the lateral walls the basic flow is unidirectional and independent of the horizontal coordinates. This has been checked experimentally by Imberger (1974). The stability of this basic flow has been analyzed by several authors including Hart (1972, 1983), Daniels et al. (1987), Drummond and Korpela (1987), Laure and Roux (1987, 1989), Kuo and Korpela (1988), Wang and Korpela (1989) and Crespo del Arco et al. (1989), whereas experimental results have been reported by Hung and Andereck (1988). Because of the application to thermal oscillations in liquid metals (Gill 1974), special interest has been taken in the case of fluids with low Prandtl number, Pr.

The basic Hadley flow can also become unstable when there is also heating from below, so that the resultant applied temperature gradient is inclined to the vertical. This situation was analyzed by Weber (1973) for the case of stress-free boundaries and small horizontal temperature gradients. His linear stability analysis was repeated by Bhattacharyya and Nadoor (1976) (for the case of rigid boundaries) and Nadoor and Bhattacharyya (1981) [for the magnetohydrodynamic (MHD) situation]. Sweet et al. (1977) used a linear, mean field approximation to investigate oscillatory convection in low Prandtl number fluids.

The restriction to small horizontal gradients was lifted by Weber (1978), but his analysis and calculations were restricted to small and moderate values of Pr. In the present article the problem is reformulated so that the case of large Pr can be dealt with. Weber worked in terms of a vertical Rayleigh number and a horizontal Grashof number, whereas in this article a vertical Rayleigh number, R_V, and a horizontal Rayleigh number, R_H, are introduced. It turns out that this has some important advantages. It enables the case of large Pr number to be treated easily, and it allows the theory to be tied in with the case of $R_V = 0$, the situation dealt with by Hart and the other authors mentioned earlier. It also allows one to appreciate the effect of increasing R_H on the shape of the basic vertical temperature profile and the other effects of increasing R_H. It turns out that the expression for the basic steady temperature distribution, which is the prime agency affecting the critical vertical Rayleigh number when Pr is large, is a function of R_H^2. (The curves in Figure 2a are nearly parabolas.) The situation for other values of Pr is more complicated.

The resulting differential equation eigenvalue problem has been solved using a low-order Galerkin method. This is convenient and is sufficiently accurate for the purpose at hand. Further, the integrals involved can be interpreted as energy integrals, and this leads to additional insight into the nature of the mechanisms involved in the instability.

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Basic flow

The situation considered is illustrated in Figure 1. It is that discussed by Weber (1978), but, with some future extensions in mind, we prefer to use some different notation. The fluid is bounded by two horizontal planes a distance d apart. A Cartesian coordinate system (x^*, y^*, z^*) is taken with origin midway between these planes and with the z^*-axis vertically upward. (The asterisks denote dimensional variables.) A linear horizontal temperature gradient is imposed in the x^*-direction, and a constant temperature difference is imposed between the two planes. Accordingly we have the temperature condition.

$$T^* = T_0 - \beta_H x^* \pm \Delta T/2 \text{ at } z^* = -(\pm d/2) \tag{2.1}$$

where β_H is a constant, which, without loss of generality, can be assumed to be positive. The ratio of the height to the length of the layer is assumed to be sufficiently small so that the motion in the horizontally central part is not affected by lateral end effects.

We treat a fluid of thermal diffusivity, κ, kinematic viscosity, ν, and density, ρ, which is given by

$$\rho = \rho_0[1 - \alpha_T(T^* - T_0)] \tag{2.2}$$

We adopt the Oberbeck-Boussinesq approximation. In nondimensional form, the governing equations can be written

$$\mathrm{Pr}^{-1}[\partial \mathbf{v}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{v}] = -\nabla p + \nabla^2\mathbf{v} + T\mathbf{k} \tag{2.3}$$

$$\partial T/\partial t + \mathbf{v}\cdot\nabla T = \nabla^2 T \tag{2.4}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{2.5}$$

Here $\mathbf{v} = (u, v, w)$ is the velocity vector, $\mathbf{k}$ is the unit vector in the z-direction and Pr is the Prandtl number ν/κ. The scales d for length, d^2/κ for time (and so κ/d for velocity), $\rho_0\kappa\nu/d^2$ for pressure (excess over hydrostatic) and $\Delta T/\mathrm{R_V}$ for the temperature (excess over the standard temperature T_0) have been chosen. The vertical Rayleigh number $\mathrm{R_V}$ is defined by

$$\mathrm{R_V} = g\alpha_T d^3\Delta T/\nu\kappa \tag{2.6}$$

The nondimensional form of the boundary conditions (for the case of rigid conducting boundaries) is

$$\mathbf{v} = 0 \text{ and } T = -(\pm \mathrm{R_V}/2) - \mathrm{R_H}x \text{ at } z = \pm 1/2 \tag{2.7}$$

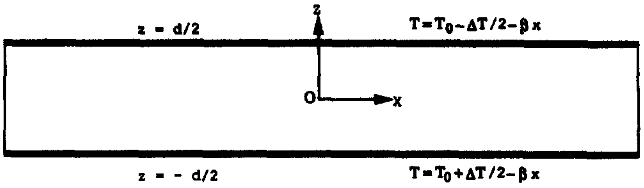

Figure 1 Definition sketch

The horizontal Rayleigh number $\mathrm{R_H}$ is defined by

$$\mathrm{R_H} = \mathrm{R_V}\, d\beta_H/\Delta T = g\alpha_T d^4\beta_H/\nu\kappa \tag{2.8}$$

Equations (2.3–2.7) admit a steady-state solution of the form

$$u_S = U(z),\ v_S = V(z),\ w_S = 0,\ p_S = p_S(x, y, z),\ T_S = \hat{T}(z) - \mathrm{R_H}x \tag{2.9}$$

provided that

$$\mathrm{D}^3 U = -\mathrm{R_H},\ \mathrm{D}^3 V = 0,\ \mathrm{D}^2\hat{T} = -\mathrm{R_H}U \tag{2.10}$$

and

$$U = V = 0 \text{ and } \hat{T} = -(\pm\mathrm{R_V}/2) \text{ at } z = \pm 1/2 \tag{2.11}$$

where D denotes the derivative d/dz.

Let $\langle(\cdot)\rangle$ denote the integral $\int_{-1/2}^{1/2}(\cdot)dz$.

The requirement that there be no net horizontal mass flux implies that

$$\langle U\rangle = 0,\ \langle V\rangle = 0 \tag{2.12}$$

The solution to Equation 2.10 subject to Equations 2.11 and 2.12 is given by

$$U(z) = \mathrm{R_H}(z/24 - z^3/6),\ V(z) = 0 \tag{2.13}$$

$$\hat{T}(z) = -\mathrm{R_V}z + \mathrm{R_H^2}(7z/5{,}760 - z^3/144 + z^5/120) \tag{2.14}$$

Stability analysis

We now perturb the steady-state solution. We write $T = T_S + \theta'$ $u = u_S + u'$, and so on. We substitute these expressions and linearize with respect to the primed quantities.

Notation

A_i, B_i, C_i	constants in Equation 4.2
d	layer depth (m)
D	differential operator, d/dz
g	gravitational acceleration (m s^{-2})
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors
l, m	dimensionless wavenumbers in the x- and y-directions
M_{ij}	matrix element
p	dimensionless pressure (excess over hydrostatic)
Pr	Prandtl number
$\mathrm{R_H}$	horizontal Rayleigh number, $g\alpha_T d^4\beta_H/\nu\kappa$
$\mathrm{R_V}$	vertical Rayleigh number, $g\alpha_T d^3\Delta T/\nu\kappa$
t	dimensionless time
$\mathbf{v}$	dimensionless velocity vector
$\hat{T}(z)$	dimensionless vertical variation of the basic temperature
ΔT	temperature difference between lower and upper boundaries (K)
$U(z), V(z)$	dimensionless steady-state velocity components
x, y, z	dimensionless Cartesian coordinates

Greek letters

α	dimensionless overall horizontal wavenumber
α_T	coefficient of volume expansion (K^{-1})
β_H	horizontal temperature gradient
θ	dimensionless temperature perturbation
κ	thermal diffusivity (m$^2\cdot$s^{-1})
ν	kinematic viscosity (m$^2\cdot$s^{-1})
ρ	density (kg$\cdot$m^{-3})
σ	dimensionless frequency for disturbances

Subscripts

c	critical
s	steady state
0	standard

Superscripts

′	perturbation quantities
*	dimensional variables

We also write

$$[u', v', w', \theta', p'] = [u(z), v(z), w(z), \theta(z), p(z)] \times \exp\{i(kx + ly - \sigma t)\} \quad (3.1)$$

When we substitute Equation 3.1 and eliminate u, v and p from the resulting perturbation equations we obtain the system

$$[\mathrm{Pr}(\mathrm{D}^2 - \alpha^2) - i(kU - \sigma)] \times (\mathrm{D}^2 - \alpha^2)w + ik\,\mathrm{D}^2Uw - \alpha^2\mathrm{Pr}\theta = 0 \quad (3.2)$$

$$[\mathrm{D}^2 - \alpha^2 - i(kU - \sigma)]\theta + \mathrm{R_H}u - w\mathrm{D}\hat{T} = 0 \quad (3.3)$$

$$[\mathrm{Pr}(\mathrm{D}^2 - \alpha^2) - i(kU - \sigma)](-\alpha^2u + ik\mathrm{D}w) + l^2\,\mathrm{D}Uw = 0 \quad (3.4)$$

subject to

$$u = w = \mathrm{D}w = \theta = 0 \text{ at } z = \pm 1/2 \quad (3.5)$$

Here $\alpha = (k^2 + l^2)^{1/2}$, the overall nondimensional wavenumber. These equations can be written as

$$\begin{bmatrix} \mathrm{L}_1 & -\alpha^2 & 0 \\ \mathrm{D}\hat{T} & \mathrm{L}_2 & -\mathrm{R_H} \\ \mathrm{L}_4 & 0 & \mathrm{L}_3 \end{bmatrix} \cdot \begin{bmatrix} w \\ \theta \\ u \end{bmatrix} = 0 \quad (3.6)$$

where

$$\mathrm{L}_1 = (\mathrm{D}^2-\alpha^2)^2 + i\mathrm{Pr}^{-1}\{\sigma(\mathrm{D}^2-\alpha^2) + k[\mathrm{D}^2U - U(\mathrm{D}^2-\alpha^2)]\} \quad (3.7)$$

$$\mathrm{L}_2 = -(\mathrm{D}^2-\alpha^2) + i(kU-\sigma) \quad (3.8)$$

$$\mathrm{L}_3 = -(\mathrm{D}^2 - \alpha^2) + i\mathrm{Pr}^{-1}(kU - \sigma) \quad (3.9)$$

$$\mathrm{L}_4 = \alpha^{-2}\{\mathrm{Pr}^{-1}l^2\,\mathrm{D}U - ik\mathrm{L}_3\mathrm{D}\} \quad (3.10)$$

Galerkin method

Weber (1978) converted the set of ordinary differential equations (Equation 3.6) into a matrix eigenvalue problem for eigenvalues σ, but we prefer to apply a Galerkin method directly to Equation 3.6, employing the standard technique described by Finlayson (1972). As trial functions (satisfying the boundary conditions) for w, θ, and u we take

$$W_i = z^{i-1}(z^2 - 1/4)^2, \quad T_i = U_i = z^{i-1}(z^2 - 1/4)^2 \quad (4.1)$$

(Finlayson showed that using these trial functions led to rapid convergence in the case of the Rayleigh-Benard problem ($\mathrm{R_H} = 0$).) We write

$$w = \sum_{i=1}^{N} A_iW_i, \quad \theta = \sum_{i=1}^{N} B_iT_i, \quad u = \sum_{i=1}^{N} C_iU_i \quad (4.2)$$

substitute into the three equations (Equation 3.6), and make the residuals orthogonal to W_i, T_i and U_i, respectively, for $i = 1, 2, \ldots, N$. Eliminating the constants A_i, B_i and C_i from the resulting homogeneous equations, we get

$$\det(M) = 0 \quad (4.3)$$

where M is the $3N \times 3N$ matrix with elements given, for $i, j = 1, 2, \ldots, N$, by

$$M_{3i-2,3j-2} = \langle \mathrm{L}_1W_j, W_i\rangle, \quad M_{3i-2,3j-1} = -\alpha^2\langle T_j, W_i\rangle, \quad M_{3i-2,3j} = 0$$

$$M_{3i-1,3j-2} = \langle \mathrm{D}T^*W_j, T_i\rangle, \quad M_{3i-1,3j-1} = \langle \mathrm{L}_2T_j, T_i\rangle, \quad M_{3i-1,3j} = -\mathrm{R_H}\langle U_j, T_i\rangle$$

$$M_{3i,3j-2} = \langle \mathrm{L}_4W_j, U_i\rangle, \quad M_{3i,j-1} = 0, \quad M_{3i,3j} = \langle \mathrm{L}_3U_j, U_i\rangle \quad (4.4)$$

Equation 4.3 may be regarded as an eigenvalue equation that yields an eigenvalue $\mathrm{R_V}$ as a function of the parameters α, Pr, $\mathrm{R_H}$, where $\alpha = (k^2 + l^2)^{1/2}$. The critical vertical Rayleigh number is the minimum of $\mathrm{R_V}$ as k and l vary.

The various integrals that appear in these equations can be expressed in terms of the quantities $E(i, j) = \langle z^{i-1}(z^2 - 1/4)^j\rangle$, which are readily evaluated (see, e.g., Finlayson (1972, p. 153). For the case $N = 2$ this process was completed algebraically, but for the case of larger N numerical computation of the integrals was performed.

Results and discussion

Qualitative results

We need to test for stability both stationary and oscillatory modes, and modes of various orientations, varying between longitudinal and transverse. The term longitudinal is applied to the situation in which the additional convective flow is in the form of rolls whose axes are parallel to the applied horizontal temperature gradient, that is, parallel to the x-axis. Longitudinal disturbances are thus independent of x and are characterized by $k = 0$. Likewise, transverse disturbances are characterized by $l = 0$.

Some general qualitative conclusions can be readily drawn from the analytical form of Equation 4.3. For $k = 0$ the equation factorizes into two equations, one factor corresponding to an even mode and the other to an odd mode. For neutral stability the frequency σ is real, and taking the real and imaginary parts of one of these equations yields two equations from which σ can be eliminated. In the first approximation the equations are sufficiently simple for the elimination to be done algebraically. We find that for the first (even) mode, we have either $\sigma = 0$ (a stationary mode) and

$$\mathrm{R_V} = \frac{28}{27\alpha^2}[(504 + 24\alpha^2 + \alpha^2)(10 + \alpha^2)] + \mathrm{R_{H^2}}\left\{\frac{17}{23{,}760} + \frac{1}{36\mathrm{Pr}(10 + \alpha^2)}\right\} \quad (5.1)$$

or (the second alternative)

$$\sigma^2 = -(10 + \alpha^2)^2\mathrm{Pr}^2 + \frac{3\mathrm{R_{H^2}}\alpha^2\mathrm{Pr}}{112\{(504 + 24\alpha^2 + \alpha^4)\mathrm{Pr} + (12 + \alpha^2)(10 + \alpha^2)\}} \quad (5.2)$$

and

$$\mathrm{R_V} = \frac{28}{27\alpha^2}(\mathrm{Pr} + 1)[504 + 24\alpha^2 + \alpha^4 + (12 + \alpha^2)(10 + \alpha^2)](10 + \alpha^2) + \mathrm{R_{H^2}}\left\{\frac{17}{23{,}760} - \frac{12 + \alpha^2}{36[(504 + 24\alpha^2 + \alpha^4)\mathrm{Pr} + (12 + \alpha^2)(10 + \alpha^2)]}\right\} \quad (5.3)$$

A short calculation shows that the right-hand side of Equation 5.3 is less than the right-hand side of Equation 5.1 if and only if $\mathrm{R_H}$ is large enough so that the right-hand side of Equation 5.2 is positive. This demonstrates the bifurcation of the neutral curve for the first longitudinal oscillatory mode from that for the first longitudinal stationary mode, at a value of $\mathrm{R_H}$, which increases as Pr increases. In particular, it is clear that unstable longitudinal oscillatory modes do not exist in the limiting case as Pr tends to infinity, because then Equation 5.2 gives no real value for σ. (This is in accord with the finding by Nield [1991] that in the corresponding problem for a porous medium, for which Darcy's law is applicable, all longitudinal oscillatory modes are stable.) In fact, for infinite Pr Equations 3.2–3.4 reduce to the pair

$$(D^2 - \alpha^2)^2 w - \alpha^2 \theta = 0 \qquad (5.4)$$

$$[D^2 - \alpha^2 - i(kU - \sigma)]\theta + (ik/\alpha^2)R_H w - D\hat{T}^* w = 0 \qquad (5.5)$$

and from the form of Equation 5.5 we can anticipate that one will get instability with $\sigma = 0$ if and only if $k = 0$. The importance of the inertial terms (in the left-hand side of Equation 2.3) for longitudinal oscillatory instability at finite Pr is clear.

We see from Equations 3.2–3.5 that the situation is generally more complicated for the transverse modes ($l = 0$) than for the longitudinal modes. For the transverse modes Equation 4.3 does not factorize into two equations, for even and odd modes. We find that the lowest transverse oscillatory mode involves a beating effect between an even and an odd mode.

When Equations 3.2–3.4 are multiplied by w, θ and u, respectively, and integrated with respect to z, we obtain

$$\Pr\langle[(D^2 w)^2 + 2\alpha^2(Dw)^2 + \alpha^4 w^4]\rangle = \alpha^2 \Pr\langle w\theta\rangle + \langle i(kU - \sigma)(D^2 - \alpha^2)w \,.\, w\rangle - ik\langle D^2 U \,.\, w^2\rangle \qquad (5.6)$$

$$\langle[(D\theta)^2 + \alpha^2\theta^2]\rangle = -\langle D\hat{T}^* w\theta\rangle - ik\langle U\theta^2\rangle + R_H\langle u\theta\rangle \qquad (5.7)$$

$$\Pr\langle[(Du)^2 + \alpha^2 u^2]\rangle = -i\langle(kU - \sigma)u^2\rangle - i(k/\alpha^2)\langle u(D^2 - \alpha^2)Dw\rangle - k^2\langle uU\, Dw\rangle - l^2\langle uw\, DU\rangle \qquad (5.8)$$

The integrals (which are of similar nature to the elements that appear in Equation 4.4) in these three equations can be interpreted as energy integrals. Those on the left-hand sides correspond to dissipation of energy, and on the right-hand sides the terms involving $i\sigma$ are growth terms, and the remainder are source terms. From Equation 5.7, for example, we see that for longitudinal modes there are terms corresponding to transfer of thermal energy through interaction of the perturbation velocity with the basic vertical and horizontal temperature gradients. For transverse modes there is also a term corresponding to the transfer of thermal energy by means of the basic horizontal velocity distribution.

Numerical results

Calculations have been performed, for various values of Pr and for a range of R_H values, of the critical value of R_V (minimized with respect to α) and the corresponding values of α and σ, and the results are presented in Figures 2–5. The presented results are confined to those for longitudinal and transverse orientations. In only one situation, of those considered, is an oblique mode favored rather than a corresponding longitudinal or transverse one. This is the case for Pr = 1 (Figure 4). In the range of R_H values from about 1,500–3,000 (where the neutral curves for longitudinal and transverse stationary modes are close together) an oblique mode can be the favored stationary mode, but the difference in the critical value of R_V is at most only 2 percent (so that, if drawn, the additional curve would have been almost coincident with those drawn); in any case, this is not physically important because an oscillatory mode is the most favored mode in this range.

The Galerkin approximation was employed with order of approximation N equal to 2 or 4. Comparison between the two sets of results showed that the $N = 2$ results for critical R_V are accurate to within 7 percent (the order of accuracy normally attainable in experiments involving Benard convection) for the parameter ranges shown (R_H from 0–6,000, R_V from 0–25,000) except for the case of transverse oscillatory modes, for which the error rises to 25 percent for the larger values of R_H. (The reason for the difference is not definitely known at present, but it is probably a result of a substantial change in the shape of the eigenfunction.) The plots presented are for values obtained with $N = 4$, that is, from computations involving the zeros of determinants of order 12. As is well known, the evaluation of determinants of large order is severely restricted by accumulated round-off error, and 12 was the largest order which could be used here. The largest value $R_H = 6{,}000$ that we have taken corresponds approximately to the maximum

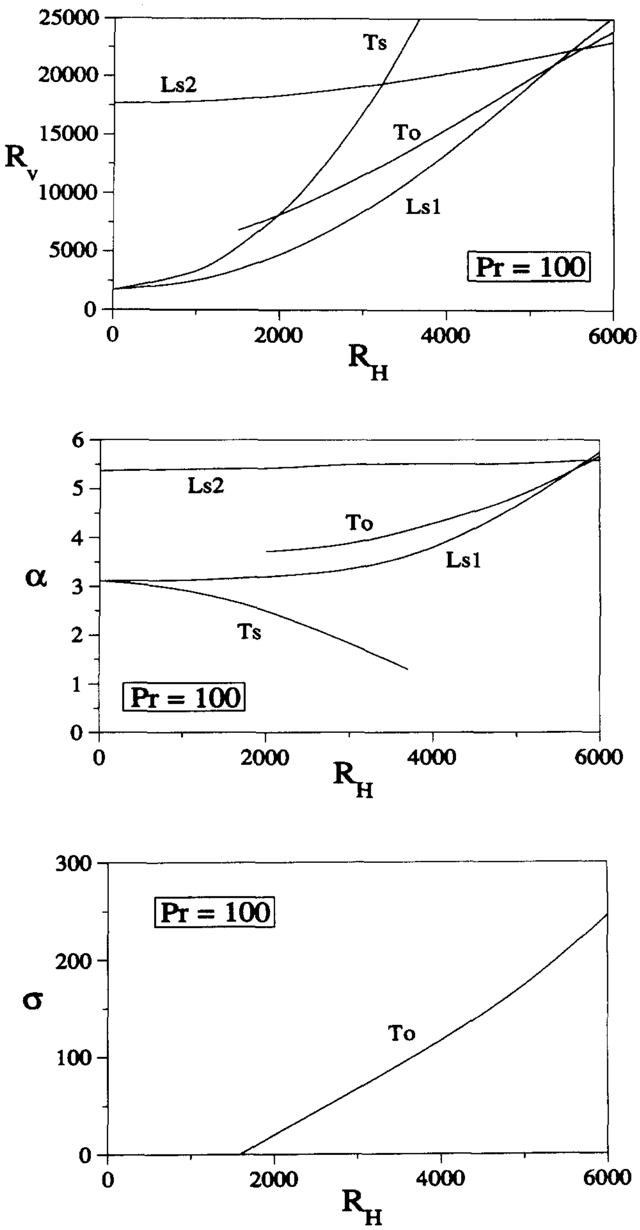

Figure 2 Values of the critical vertical Rayleigh number R_V, and the corresponding horizontal wavenumber α and frequency σ, for Pr = 100. In this and the following figures, the labels for the various modes are as follows: Ls1, first (even) longitudinal stationary; Ls2, second (odd) longitudinal stationary; Lo, longitudinal oscillatory; Ts, transverse stationary; To, transverse oscillatory

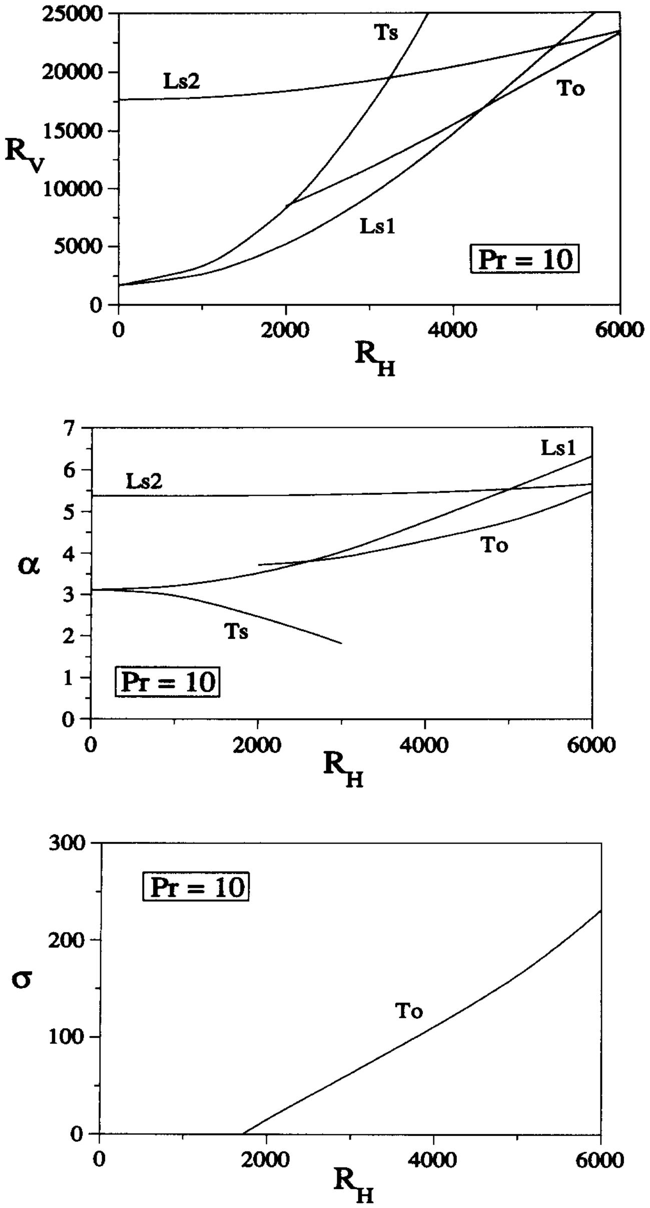

Figure 3 Values of the critical vertical Rayleigh number R_V, and the corresponding horizontal wavenumber α and frequency σ, for Pr = 10

value for which the basic vertical temperature gradient is negative throughout the layer. At higher values of R_H our Galerkin approximation breaks down because the eigenfunction for w becomes markedly different in form from the polynomial trial functions that we have chosen. This is not surprising because one would expect that when $D\hat{T}^*$ becomes positive for a range of z (in the middle of the layer) the asymptotic form (as R_H tends to infinity) of w will be oscillatory in that range. This implies that the form of the perturbation flow at large values of R_H will be in the form of multiple layers of superposed rolls. (At sufficiently large R_H the Boussinesq approximation will not be valid.)

Figures 2a and 3a show that for large Pr values (10 and larger) an even longitudinal stationary mode is favored (the most unstable) for values of R_H up to about 5,000, after which a transverse oscillatory mode is favored within a small range of R_H values, beyond which an odd longitudinal stationary mode (a two-layered roll pattern) is favored. In contrast (see Figure 4a), for Pr = 1 a transverse stationary mode is favored for small values of R_H and a longitudinal mode is favored at larger values of R_H. In each of these cases the critical R_V increases as R_H increases within the range shown. (The results of Hart [1972] indicate that this trend does not continue for higher values of R_H.) The situation for Pr = 0.1 is dramatically different (see Figure 5a). Now the transverse stationary mode is favored for all R_H values, and R_V decreases as R_H increases, reaching a zero value (and then going negative) at R_H = 1,000. Thus, even in the absence of an applied vertical temperature gradient, the Hadley flow becomes unstable when R_H is sufficiently large. This result for Pr = 0.1 is in accord with the findings of Hart (1972). An estimate from his Figure 5 is R_H = 925 when Pr = 0.1.

The corresponding critical horizontal wavenumber is plotted in Figures 2b, 3b, 4b and 5b. For the longitudinal stationary

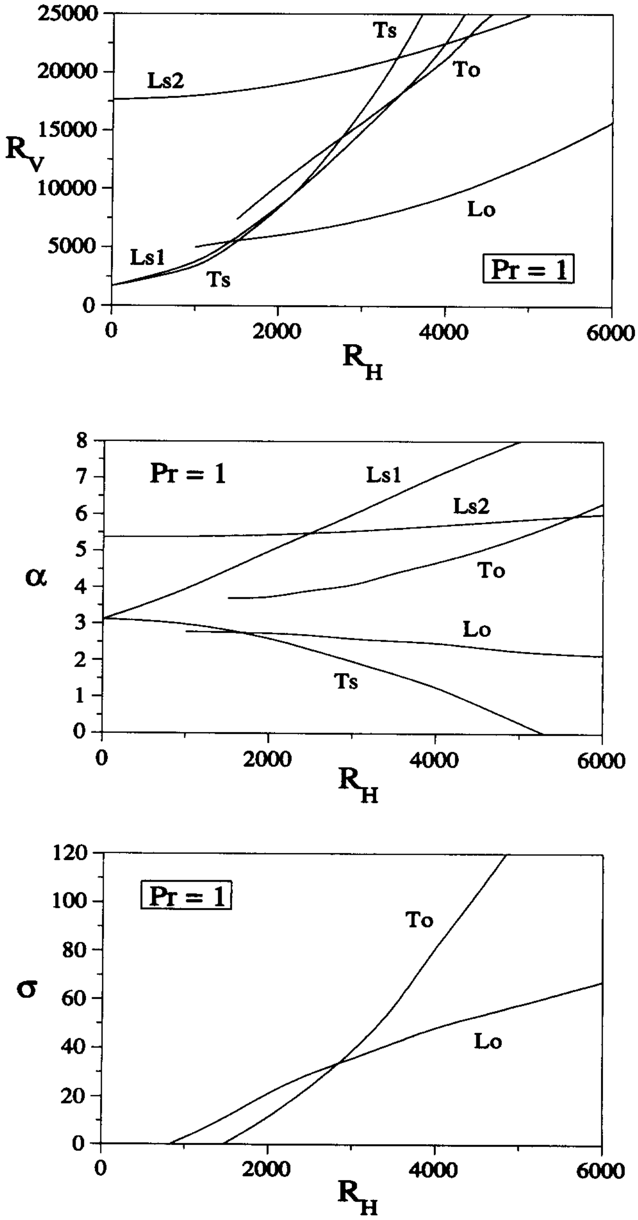

Figure 4 Values of the critical vertical Rayleigh number R_V, and the corresponding horizontal wavenumber α and frequency σ, for Pr = 1

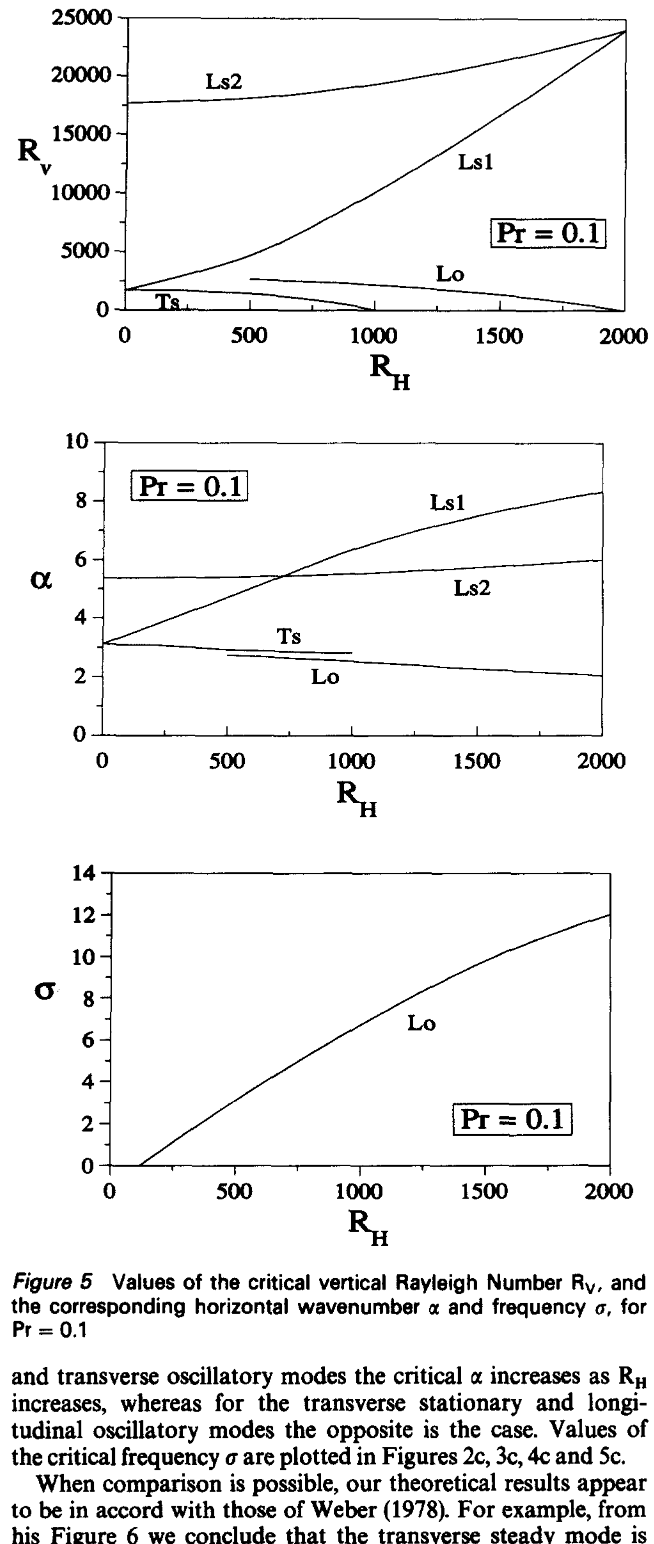

Figure 5 Values of the critical vertical Rayleigh Number R_V, and the corresponding horizontal wavenumber α and frequency σ, for Pr = 0.1

and transverse oscillatory modes the critical α increases as R_H increases, whereas for the transverse stationary and longitudinal oscillatory modes the opposite is the case. Values of the critical frequency σ are plotted in Figures 2c, 3c, 4c and 5c.

When comparison is possible, our theoretical results appear to be in accord with those of Weber (1978). For example, from his Figure 6 we conclude that the transverse steady mode is favored when Pr = 1, and an estimate of 16.0 for $R_2 \times 10^4$, yielding a value of 3,310 for R_V, is obtained. The present calculations gave the value 3,333.

We conclude that our approach is giving useful results for R_H values up to 6,000. The range of higher R_H values remains to be explored. As was mentioned earlier, the information at hand suggests that the eigenfunction changes dramatically, there being an increasing number of zeros within the layer, as R_H increases.

As far as the author is aware there are no experimental results available for comparison.

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